

# Comparison of Motion Control Techniques for a 3RPS Parallel Manipulator

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**Abstract**— In this paper, three motion control techniques are discussed for 3RPS parallel robot. The main purpose of this research is to study the model-based control techniques to determine whether these techniques are capable of reducing the tracking error considering the measurement noise and external disturbance. In order to evaluate the proposed approach, three different techniques are presented. Inverse dynamics control that is a simple controller and robust Inverse dynamics control and adaptive Inverse dynamics control which are other expansions of Inverse dynamics control. Dynamic modeling of the manipulator has been analyzed in order to be used for both simulation and control purposes. Then the structure of the controllers and their features are discussed. Performance of the controllers has been examined through simulation. It is observed that the adaptive Inverse dynamics control is capable of providing suitable motion tracking, while high amount of uncertainties in model is considered.

**Keywords**—inverse dynamics control; robust inverse dynamics control; adaptive inverse dynamics control; 3RPS parallel robot.

## I. INTRODUCTION

Robots and especially parallel robots have improved human life in the past century. Parallel robots are fast and have light weight, heavy load capacity and high stiffness. These characteristics makes them great candidates to be applied in factories and laboratory researches. They are also being used in sensing, flight simulators, electronic gaming machines and also medical applications.

This paper focuses on the 3RPS parallel robot; this robot was first proposed by Hunt [1]. Then it was applied for various different applications. One of the most successful applications of 3RPS is in rehabilitation and arm exoskeleton and 3RPS parallel robot is used as robot's wrist in this regard [2]. Other applications of 3RPS includes studying on 3RPS parallel robot as ARTISAN robots end effector [3] electronic gaming machines and flight simulation.

Parallel robots are designed for two different types of application. In the first one the end-effector of the robot must follow a transfer and rotational path in a specified time while no interacting forces are being made with the environment through moving platform of the robot. For the purpose of obtaining such aim, several controlling strategies such as decentralized PD Control, feed forward control [4], inverse

dynamics control [5], robust inverse dynamics control [6], and adaptive control [7] are discussed. A survey on Control of Parallel Manipulator can be found in [8]. The second application consists of a status in which the moving platform is in connection with the environment [9].

Since the robot is time-varying and nonlinear system with model uncertainties like load variations, friction and external disturbances, motion control of robot manipulator is a difficult mission [10]. When upper bounds of uncertainties is known, robust control methods is applicable to robot manipulator. This method is capable to rejection of disturbance and measurement noise and it can improve the performance of system [11]. In recent papers [12-15], some robust control strategies have been instituted for robot manipulators. Spong [16] proposed a robust control strategy for robot manipulators with uncertainty bounds. Wang [17] was suggested a robust control law using auxiliary polynomials in position and velocity tracking errors. An approximate Jacobian feedback control law for set point control of a robot with uncertainties in the entire kinematics and Jacobian matrix from joint space to task space and in the dynamics was proposed [18]. A task space robust control scheme with suitable tracking performance under the imperfect transformation was suggested [19]. Also, adaptive control methods have been discussed in [20, 21].

The focus of this paper is application of several motion control techniques based on inverse dynamics control. The goal is to follow the moving platform of the 3RPS parallel robot based on a desired trajectory. Having a completely free moving platform without applying any external forces considered as position control.

The paper is organized as follows: Section II gives a description of the 3-RPS robot. Section III presents dynamic formulation of the robot. Section IV explains the Motion Control Techniques and its features and finally the simulations are presented in section V.

## II. DESCRIPTION OF THE 3-RPS ROBOT

3RPS mechanism consists of a moving platform and a fixed platform, three identical limbs connect these two platforms. Fig. 1 depicts the 3RPS structure. Each limb consists of a prismatic joint, a spherical joint at the top and a revolute joint which is connected to the base. The three axis of the revolute

joints at the base are arranged in 120 degrees and the axis of every one is parallel to the opposite segment of the triangular base. The structure comes up with one translational and two rotational degrees of freedom, and is driven by three linear actuators.

Fig.2 demonstrates the spatial 3-RPS parallel robot geometry. As it can be perceived from the figure, two

Cartesian coordinate systems  $A(x, y, z)$  and  $B(u, v, w)$  are attached to the fixed base and moving platform, respectively. The transformation of the center of the moving platform P, is denoted by  $P = [p_x, p_y, p_z]$ , and a  $3 \times 3$  rotation matrix  ${}^A R_B$ . Both  $\Delta A_1 A_2 A_3$  and  $\Delta B_1 B_2 B_3$  are equilateral triangles with  $OA_1 = OA_2 = OA_3 = g$  and  $pB_1 = pB_2 = pB_3 = h$ .

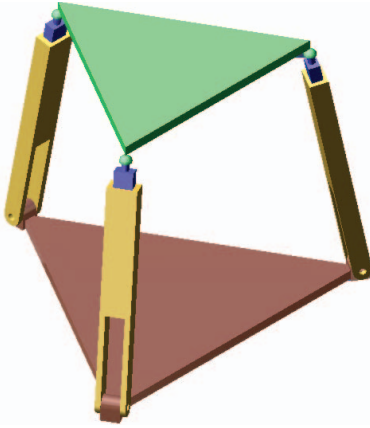


Fig. 1. The 3RPS parallel robot

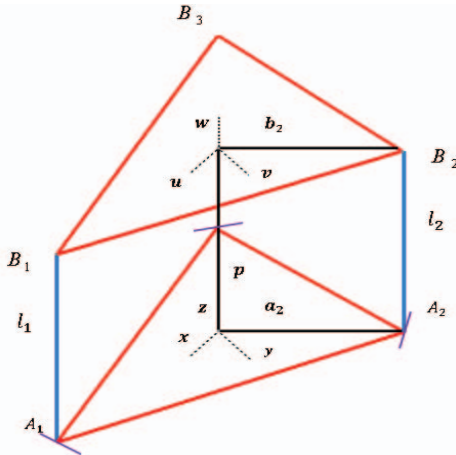


Fig. 2. The geometry of the spatial 3-RPS parallel robot[22]

### III. DYNAMICS ANALYSIS

Dynamics of the 3RPS parallel robot was extensively presented in [22]. In this section dynamic analysis of 3RPS parallel robot is presented briefly. The dynamic equations of 3RPS parallel robot can be written in an explicit form of:

$$\mathbf{M}(\boldsymbol{\chi})\ddot{\boldsymbol{\chi}} + \mathbf{C}(\boldsymbol{\chi}, \dot{\boldsymbol{\chi}})\dot{\boldsymbol{\chi}} + \mathbf{G}(\boldsymbol{\chi}) = \mathbf{F} \quad (1)$$

In which,  $\boldsymbol{\chi}$  is a vector of the generalized coordinates that is corresponding to each given manipulator location,  $\mathbf{M}(\boldsymbol{\chi})$  denotes the system mass matrix,  $\mathbf{C}(\boldsymbol{\chi}, \dot{\boldsymbol{\chi}})$  denotes the Coriolis and centrifugal terms matrix,  $\mathbf{G}(\boldsymbol{\chi})$  denotes the gravity vector, and  $\mathbf{F}$  denotes the generalized force. The dynamic matrixes of 3RPS parallel robot can be determined by:

$$\begin{aligned} \mathbf{M}(\boldsymbol{\chi}) &= \mathbf{M}_P + \sum_{i=1}^{i=3} \mathbf{M}_{li}; \mathbf{C}(\boldsymbol{\chi}, \dot{\boldsymbol{\chi}}) = \mathbf{C}_P + \sum_{i=1}^{i=3} \mathbf{C}_{li} \\ \mathbf{G}(\boldsymbol{\chi}) &= \mathbf{G}_P + \sum_{i=1}^{i=3} \mathbf{G}_{li}, \end{aligned} \quad (2)$$

where  $\mathbf{M}_{li}$ ,  $\mathbf{C}_{li}$  and  $\mathbf{G}_{li}$  are the dynamic matrixes of limbs and expressed by the following equations.

$$\begin{aligned} \mathbf{M}_{li} &= \mathbf{J}_i^T \mathbf{M}_i \mathbf{J}_i; \mathbf{C}_{li} = \mathbf{J}_i^T \mathbf{M}_i \dot{\mathbf{J}}_i + \mathbf{J}_i^T \mathbf{C}_i \mathbf{J}_i \\ \mathbf{G}_{li} &= \mathbf{J}_i^T \mathbf{G}_i \\ \mathbf{M}_i &= m_{i2} \hat{\mathbf{s}}_i \hat{\mathbf{s}}_i^T - \frac{1}{l_i^2} \mathbf{I}_{xxi} \hat{\mathbf{s}}_{ix}^2 - m_{ce} \hat{\mathbf{s}}_{ix}^2 \\ \mathbf{C}_i &= -\frac{2}{l_i} m_{co} \dot{l}_i \hat{\mathbf{s}}_{ix}^2 - \frac{1}{l_i^2} m_{i2} c_{i2} \hat{\mathbf{s}}_i \dot{x}_i^T \hat{\mathbf{s}}_{ix}^2 \\ \mathbf{G}_i &= (m_{ge} \hat{\mathbf{s}}_{ix}^2 - m_{i2} \hat{\mathbf{s}}_i \hat{\mathbf{s}}_i^T) \mathbf{g} \end{aligned} \quad (3)$$

in which

$$\begin{aligned} m_{co} &= \frac{1}{l_i} m_{i2} c_{i2} - \frac{1}{l_i^2} (\mathbf{I}_{xxi} + l_i^2 m_{ce}) \\ m_{ge} &= \frac{1}{l_i} (m_{i1} c_{i1} + m_{i2} (l_i - c_{i2})) \\ m_{ce} &= \frac{1}{l_i^2} (m_{i1} c_{i1}^2 + m_{i2} c_{i2}^2) \end{aligned} \quad (4)$$

and the dynamic matrixes of moving platform are given by the following relation:

$$\begin{aligned} \mathbf{M}_P &= \begin{bmatrix} m \mathbf{I}_{3 \times 3} & \mathbf{0}_{3 \times 3} \\ \mathbf{0}_{3 \times 3} & {}^A \mathbf{I}_P \end{bmatrix}; \mathbf{C}_P = \begin{bmatrix} \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} \\ \mathbf{0}_{3 \times 3} & \boldsymbol{\omega} \times {}^A \mathbf{I}_P \end{bmatrix}, \\ \mathbf{G}_P &= \begin{bmatrix} -mg \\ \mathbf{0}_{3 \times 1} \end{bmatrix} \end{aligned} \quad (5)$$

where  $l_i$ ,  $\hat{\mathbf{s}}_i$ , and  $\boldsymbol{\omega}$  are considered at the limb lengths, the screw axis, and angular velocity of moving platform respectively. The geometrical and inertial parameters of the manipulator are given in table 1.

Note that three of the position and orientation parameters of moving platform are dependent and they are derived from kinematic analysis.

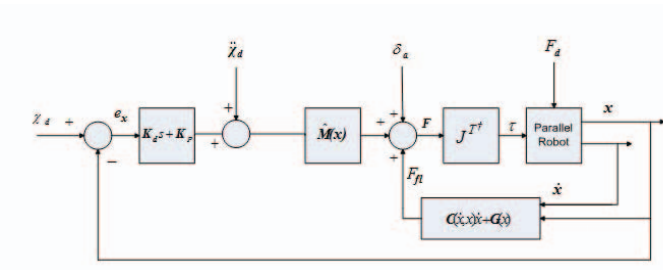


Fig. 3. Robust inverse dynamics control scheme(adopted from[9])

$$\chi = \begin{bmatrix} p_x \\ p_y \\ p_z \\ \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} f(\phi, \theta, p_z) \\ f(\phi, \theta, p_z) \\ p_z \\ \alpha \\ \beta \\ f(\phi, \theta, p_z) \end{bmatrix} \quad (6)$$

projection from Cartesian space to joint space is performed by pseudo inverse of Jacobian transpose,  $J^{T^\dagger}$ , as:

$$\tau_{\min} = J^{T^\dagger} F, \quad (7)$$

where  $\tau_{\min}$  is The minimum norm solution for actuator forces.

TABLE I. GEOMETRIC AND INERTIA PARAMETERS OF THE 3RPS PARALLEL ROBOR

Description	Quantity
$R_a$ : Fixed base radius	1.2 m
$R_b$ : Moving platform radius	1.2 m
$m$ : Moving platform mass	1150kg
$m_{i1}$ : Cylinder mass	85kg
$m_{i2}$ : Piston	22kg
$c_{i1}$ : Cylinder center of mass	0.245m
$c_{i2}$ : Piston center of	0.245m
${}^A I_P$ : Moving platform moment of inertia	Diag(570,285,285) $kg.m^2$
${}^A I_{ci1}$ : Cylinder moment of inertia	Diag(16,16,0) $kg.m^2$
${}^A I_{ci2}$ : Piston moment of inertia	Diag(4,4,0) $kg.m^2$

#### IV. MOTION CONTROL TECHNIQUES

In this section, three techniques of motion control are described for parallel robots. All of the presented techniques in this paper are based on invers dynamics control and were extensively presented in [9].

##### A. Dynamics Control

Inverse Dynamics Control (IDC) is a general and effective method in controlling robots. This method is considered as a

starting point for developing various advanced robotics controllers. In Inverse Dynamics Control, non-linear dynamic model is added as a corrective term to PD controller. Using this method, non-linear behavior of the robot will be weakened considerably and as a result, the performance of linear control will be improved significantly. The control effort of such structure is calculated by[9]:

$$\begin{aligned} \hat{M}(\chi)a + \hat{C}(\chi, \dot{\chi})\dot{\chi} + \hat{G}(\chi) &= F \\ a &= \ddot{x}_d + K_d \dot{e}_x + K_p e_x \end{aligned} \quad (8)$$

Note that the notion  $(\hat{\cdot})$ , represents the estimated value of  $(\cdot)$  terms, while  $k_d$ , and  $k_p$ , denote the derivative and proportional controller gains, respectively and motion error vector  $e_x$  denotes.

##### B. robust inverse dynamics Control

Despite the fact that inverse dynamics control has indicated appropriate performance in pursuing the desired trajectory, it has some defects as well. For instance, inverse dynamics control is not a good option to model uncertainty. The general aim in model-based controllers is to convert the non-linear dynamic equation into linear using feedback. However, because of different uncertainties sources in the system such as unmodeled dynamics, unknown parameters, calibration errors, and unknown disturbance wrenched it is almost impossible [9]. The robust inverse dynamics control will improve the defects of this controller by adding a robustifying corrective term to the inverse dynamics control structure.

In the presented controlling structure in Fig. 3,  $\delta_a$  is robustifying controlling input and is added to the model for the purpose of compensating the modeling uncertainties. This term is defined using (9)[9].

$$\delta_a = \begin{cases} -\rho \frac{\mathbf{v}}{\|\mathbf{v}\|} & \text{if } \|\mathbf{v}\| > e \\ -\rho \frac{\mathbf{v}}{e} & \text{if } \|\mathbf{v}\| \leq e \end{cases} \quad (9)$$

in witch  $e$  is a smoothing threshold and  $\mathbf{v}$  expressed by the following equations.

$$\begin{aligned} \mathbf{v} &= \mathbf{B}^T \mathbf{P} \boldsymbol{\varepsilon} \\ \boldsymbol{\varepsilon} &= \begin{bmatrix} \chi - \chi_d \\ \dot{\chi} - \dot{\chi}_d \end{bmatrix}, \mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{K}_p & -\mathbf{K}_d \end{bmatrix}, \mathbf{B} = \begin{bmatrix} \mathbf{0} \\ \mathbf{I} \end{bmatrix} \end{aligned} \quad (10)$$

Moreover, matrix  $\mathbf{P}$  is the symmetric positive definite function derived from matrix Lyapunov equation given in (11) for any arbitrary symmetric positive definite matrix  $\mathbf{Q}$ .

$$\mathbf{A}^T \mathbf{P} + \mathbf{P} \mathbf{A} = -\mathbf{Q} \quad (11)$$

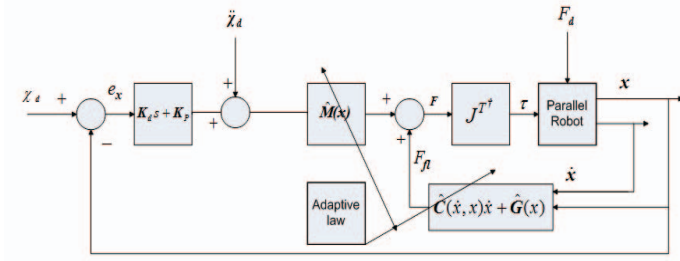


Fig. 4. Adaptive inverse dynamics control scheme(adopted from [9])

### C. Adaptive Inverse Dynamics Control

Adaptive Inverse Dynamics Control is another expansion of IDC control. In an adaptive controller, the controller parameters are adjusted by updating dynamics matrices. Adaptive control try to eliminate the difference between precision values of dynamic matrices and their estimates. The error dynamics in Adaptive Inverse Dynamics is[9],

$$A = \begin{bmatrix} 0 & I \\ -K_p & -K_d \end{bmatrix}, B = \begin{bmatrix} 0 \\ I \end{bmatrix}, \Phi = \hat{M}^{-1}Y(\mathcal{X}, \dot{\mathcal{X}}, \ddot{\mathcal{X}}) \quad (13)$$

by using the following Lyapunov function,

$$V = \epsilon^T P \epsilon + \tilde{\theta}^T \Gamma \theta, \quad (14)$$

The adaptation law is derived for updates.

$$\dot{\tilde{\theta}} = \Gamma^{-1} \Phi^T B^T P \epsilon \quad (15)$$

Fig. 4 depicts the structure of adaptive inverse dynamics control in task space

## V. SIMULATION

In this section it is aimed to examine tracking performance of the system, while different motion control approaches are applied to the 3RPS parallel manipulator. In the first simulation, %50 perturbation in all kinematics and inertial parameters are considered. In order to examine robot behavior in high acceleration motion, the whole maneuver is traversed in only 2 (s). Controller gains are set to  $k_d = 10^3 \text{diag}[1,1,2,1,1,1]$  and  $k_p = 10^2 \text{diag}[3,3,3,3,3,3]$ . Moreover, the robust controller parameters, are set to  $e = 0.5$ ,  $\rho = 200$ . Note that the performance of controllers is examined in presence of measurement noise and external disturbance. A 10 KN force step disturbance in x, y and z directions and a torque step disturbance in x, y and z directions is applied at time 0.6 seconds. The amplitude of noise is about  $10^{-4}$  times the peak values of the measurement signal.

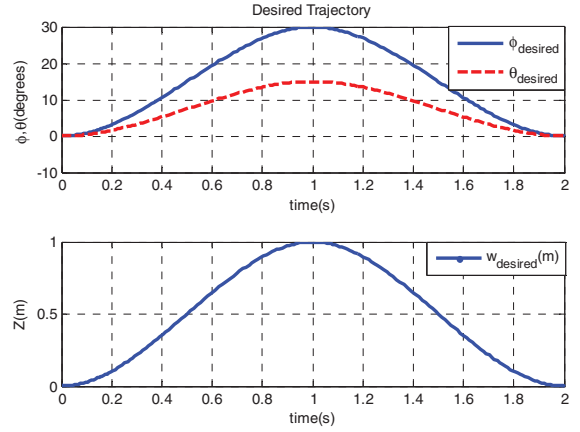


Fig. 5. The desired trajectories of 3RPS moving platform

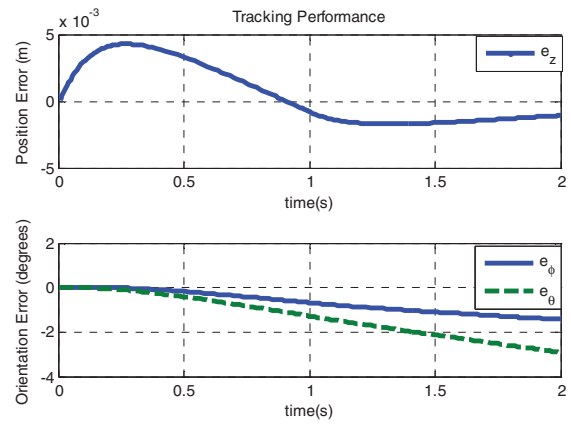


Fig. 6. The closed loop tracking performance of 3RPS parallel robot; inverse dynamics control(IDC).

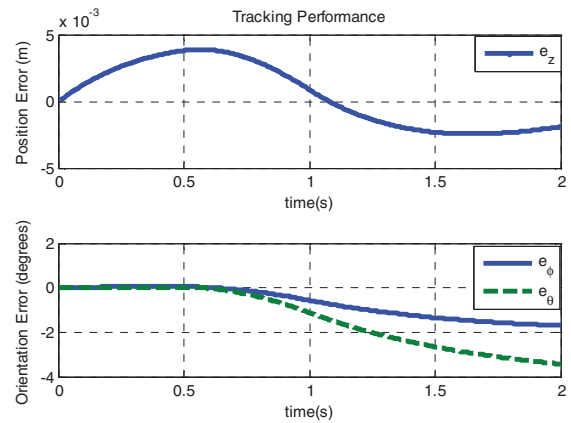


Fig. 7. The closed loop tracking performance of 3RPS parallel robot; robust inverse dynamics control.

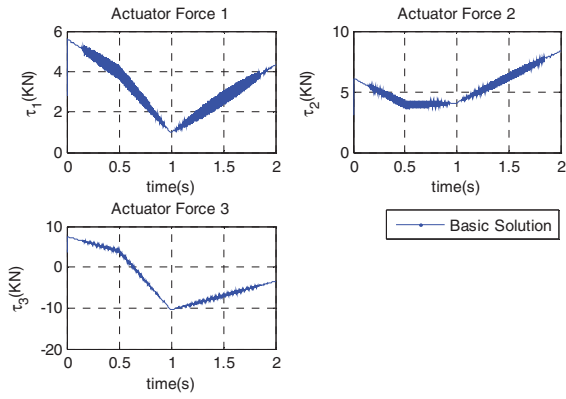


Fig. 8. The actuator forces of 3RPS parallel robot; robust inverse dynamics

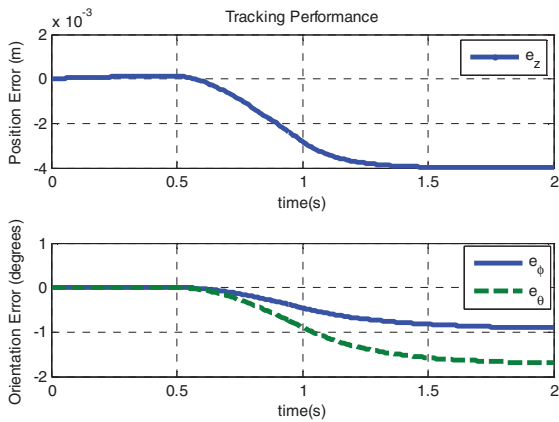


Fig. 9. The closed loop tracking performance; adaptive inverse dynamics control.

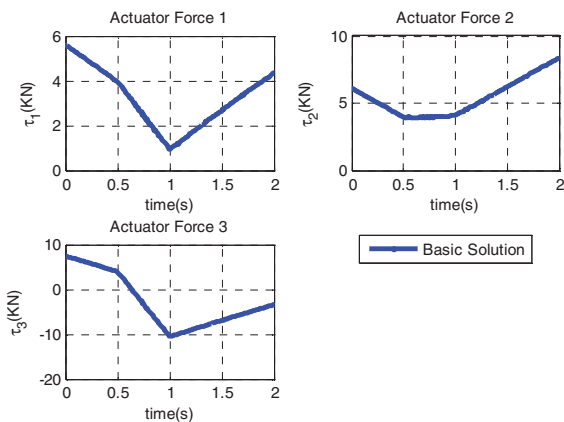


Fig. 10. The actuator forces of 3RPS parallel robot; adaptive inverse dynamics control.

Fig. 5 shows the desired trajectory of the moving platform. The tracking errors for IDC, robust IDC, and adaptive IDC in presence of measurement noise and external disturbance are illustrated in Fig. 6, Fig.7, and Fig.9 respectively. In order to make comparison between controllers, no changes are considered in the PD controller gains. Considering the

simulation results, it is observed that, the closed loop tracking performance of the 3RPS manipulator with adaptive IDC control is the most suitable. By comparing Fig. 6 with Fig. 7, it can be seen that, by using robust control the accuracy of tracking errors are not changed that much, however, the robust IDC is capable to reduce the sharp variations in the error experienced by regular IDC. However, the difference between tracking error in the robust IDC and the adaptive IDC is considerable. The required actuator forces of robust IDC and adaptive IDC are revealed in Fig. 8, and Fig. 10, respectively.

The effect of measurement noise and external disturbance is clearly seen in the response. The abrupt change in the tracking error at time 0.6 seconds shows that the controllers are not able to completely remove the effects of disturbances on the system.

## VI. CONCLUSION

In this paper, several model-based control techniques were introduced for 3RPS parallel robot. As explained before, the overall system performance depends on the precision of dynamics matrices information; that is the reason the dynamic model was presented. In the simulation section, controllers performance was examined having external disturbance, measurement noise, and %50 perturbation in all kinematics and inertial parameters. The results of the simulations demonstrates the ability of the adaptive IDC control to track a desired trajectory. Moreover, using IDC control as the motion control; the tracking errors rate did not changed significantly. But the robust IDC is able to decrease the error sharp variations are observed for IDC. However, the controllers are not capable of eliminating disturbances completely. Finally some robust adaptive control strategies are suggested to solve the defects of robust and adaptive control methods while the advantages of them is protected.

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